

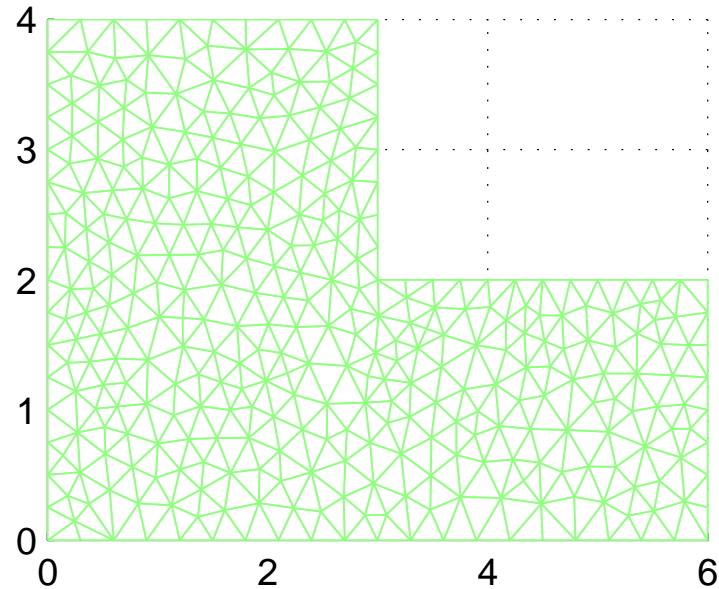
Numerical results

Patrick Ciarlet, Errell Jamelot

Laboratoire de Mathématiques Appliquées - ENSTA

Characteristics of the meshes

● The model domain

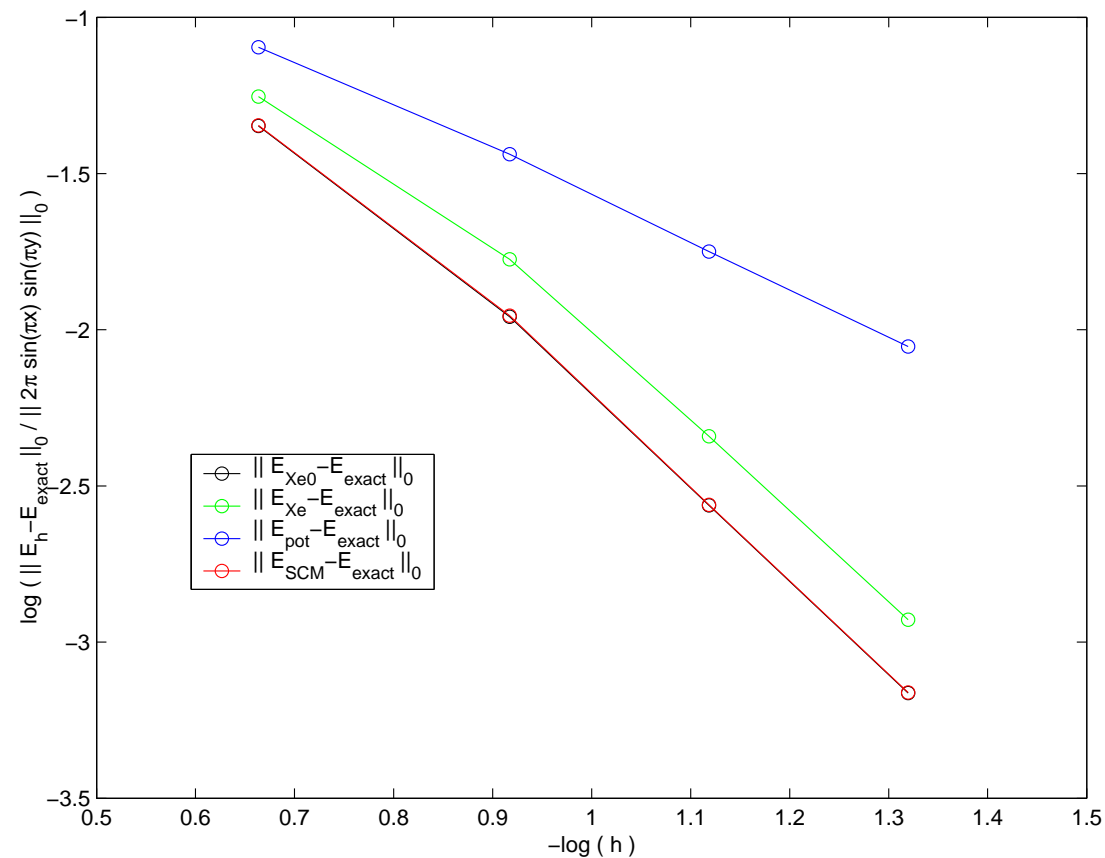


Mesh	1	2	3	4
Number of triangles	620	2780	9920	39680
Number of nodes	348	1315	5109	20137

Regular case, $L^2(\omega)$ norms

• $\operatorname{div} \vec{E} = 2\pi \sin(\pi x) \sin(\pi y)$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$,

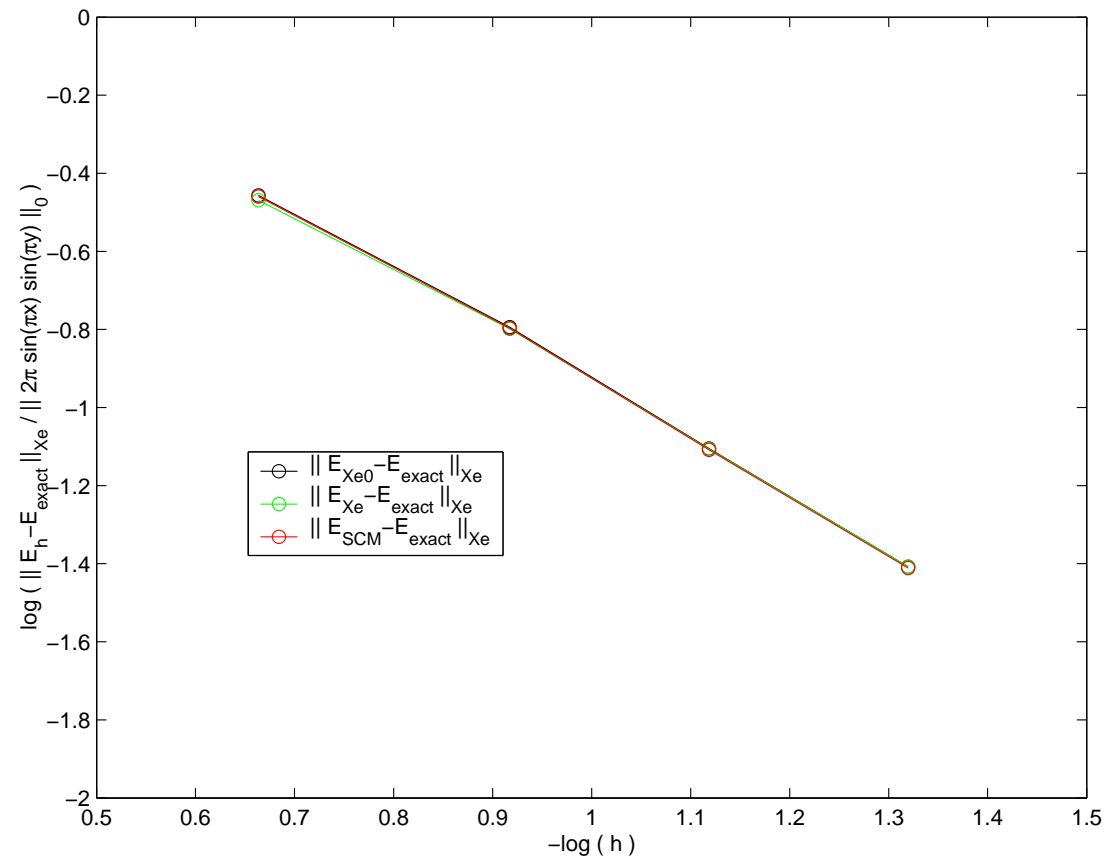
$$\vec{E}_{exact}(x, y) = - \begin{pmatrix} \cos(\pi x) \sin(\pi y) \\ \sin(\pi x) \cos(\pi y) \end{pmatrix}, \quad \|2\pi \sin(\pi x) \sin(\pi y)\|_0 = 4.16 \times 10^1.$$



Regular case, $X_{\vec{E}}$ norms

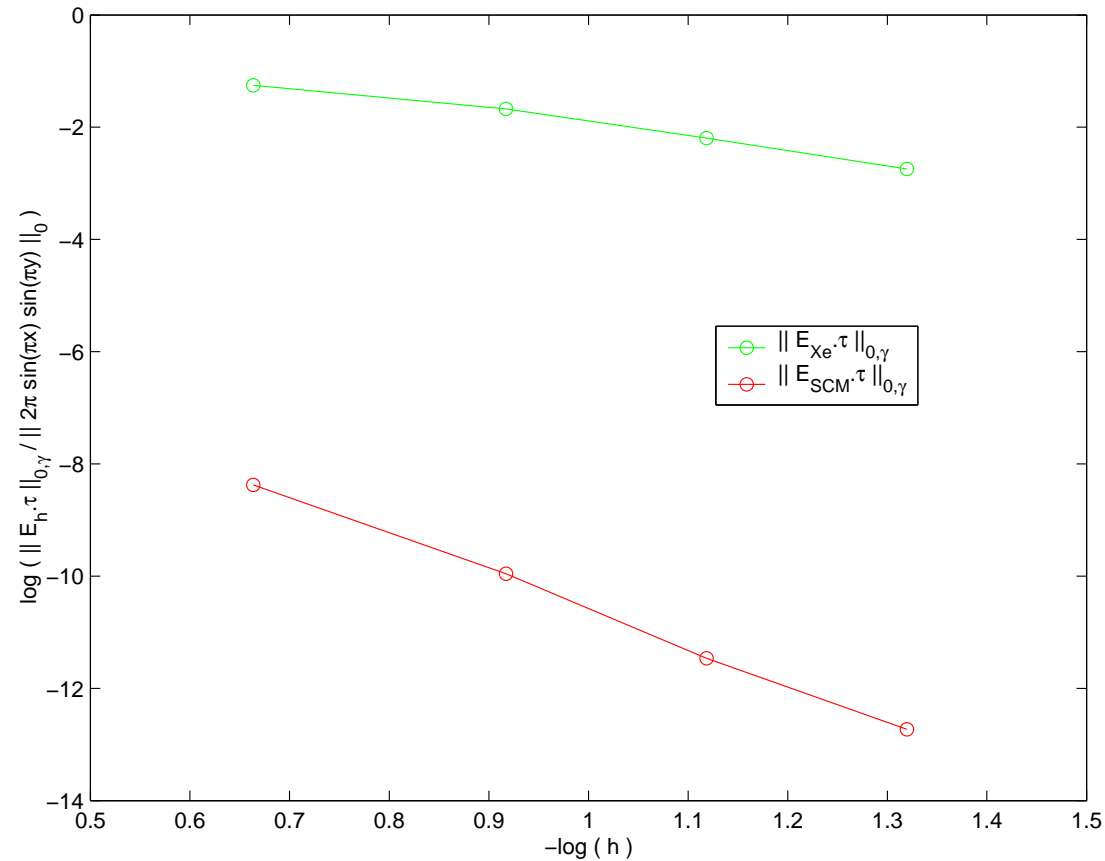
● $\operatorname{div} \vec{E} = 2\pi \sin(\pi x) \sin(\pi y)$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$,

$$\|\vec{v}\|_{X_{\vec{E}}}^2 = \|\operatorname{div} \vec{v}\|_0^2 + \|\operatorname{curl} \vec{v}\|_0^2 + \int_{\gamma} (\vec{v} \cdot \vec{\tau})^2 d\gamma.$$



Regular case, $L^2(\gamma)$ norms

• $\operatorname{div} \vec{E} = 2\pi \sin(\pi x) \sin(\pi y)$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$.

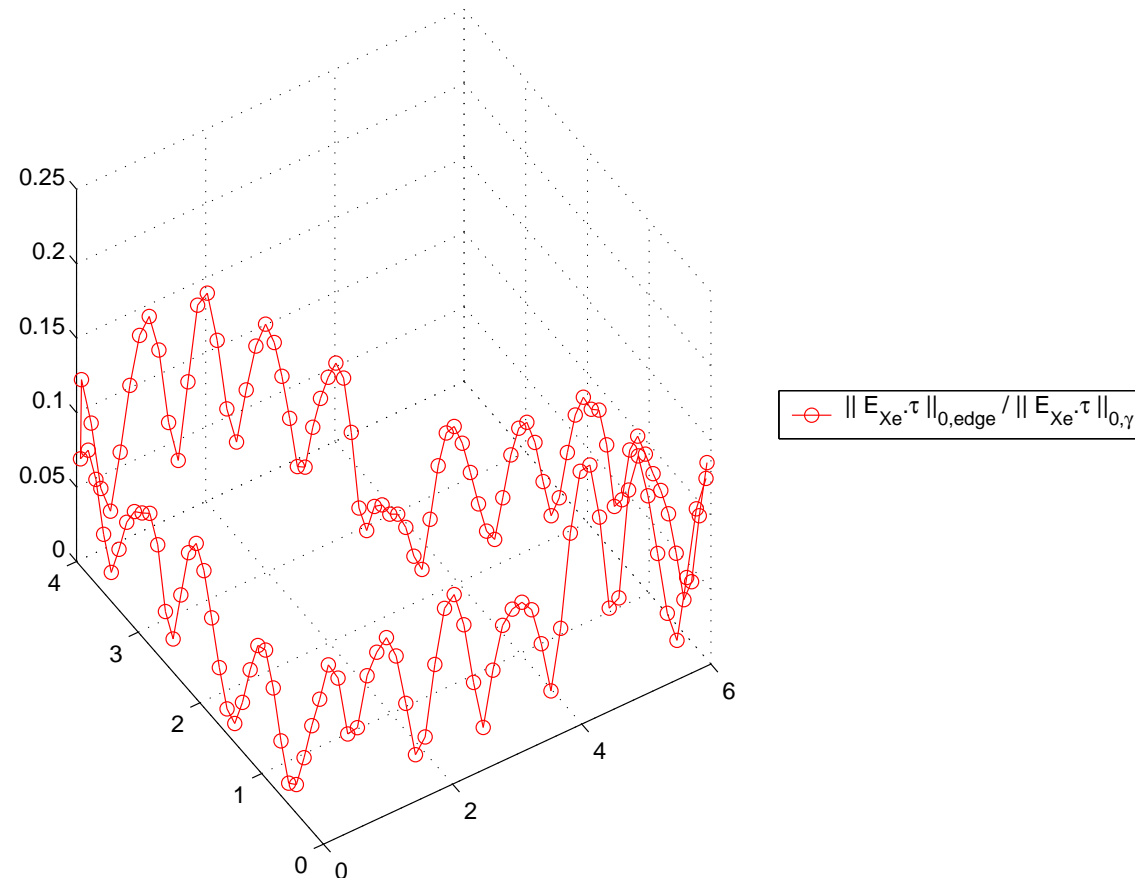


Regular case, tangential component of \vec{E} on γ

● $\operatorname{div} \vec{E} = 2\pi \sin(\pi x) \sin(\pi y)$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$,

$$\|\vec{E}_{X_{\vec{E}}} \cdot \vec{\tau}\|_{0,\gamma} = 8.65 \times 10^{-1} ,$$

$$\|\vec{E}_{X_{\vec{E}}} \cdot \vec{\tau}\|_{0,\gamma} / \|2\pi \sin(\pi x) \sin(\pi y)\|_0 = 2.08 \times 10^{-2} \text{ on the second mesh.}$$

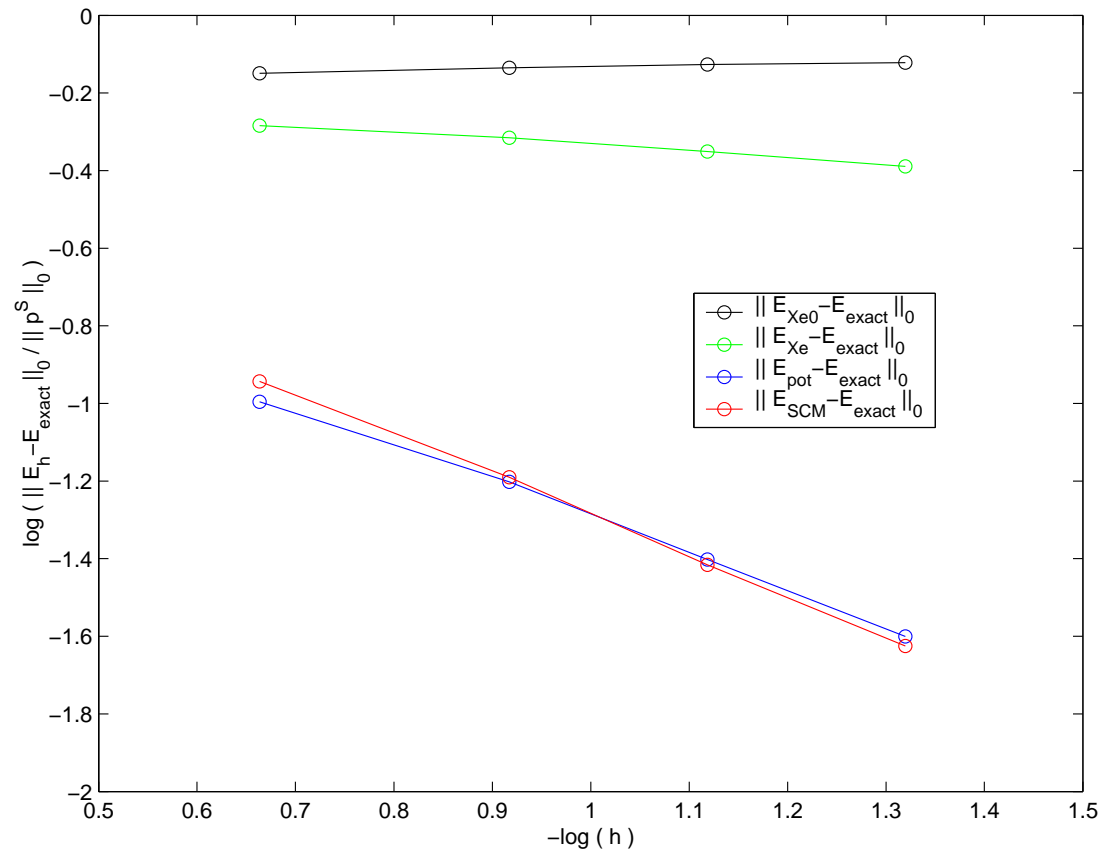


Singular case, $L^2(\omega)$ norms

• $\text{div } \vec{E} = p^S$, $\text{curl } \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_\gamma = 0$, $\vec{E}_{exact} = -\text{grad } \phi^S$, with :

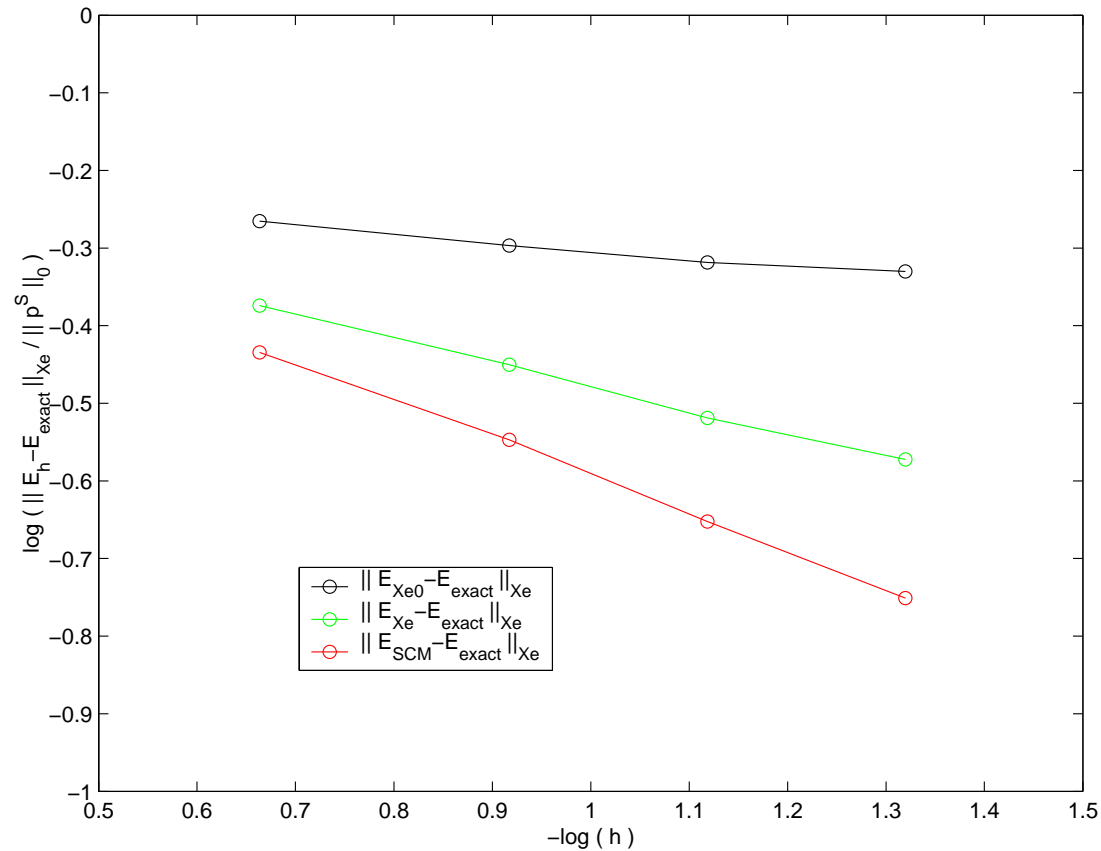
$$p^S = \tilde{p}_{H^1} + r^{-\frac{2}{3}} \sin\left(\frac{2}{3}\theta\right), \quad \phi^S = \tilde{\phi}_{H^2} + \beta r^{\frac{2}{3}} \sin\left(\frac{2}{3}\theta\right), \quad \beta = \|p^S\|_0^2 / \pi.$$

$$\|p^S\|_0 = 1.87.$$



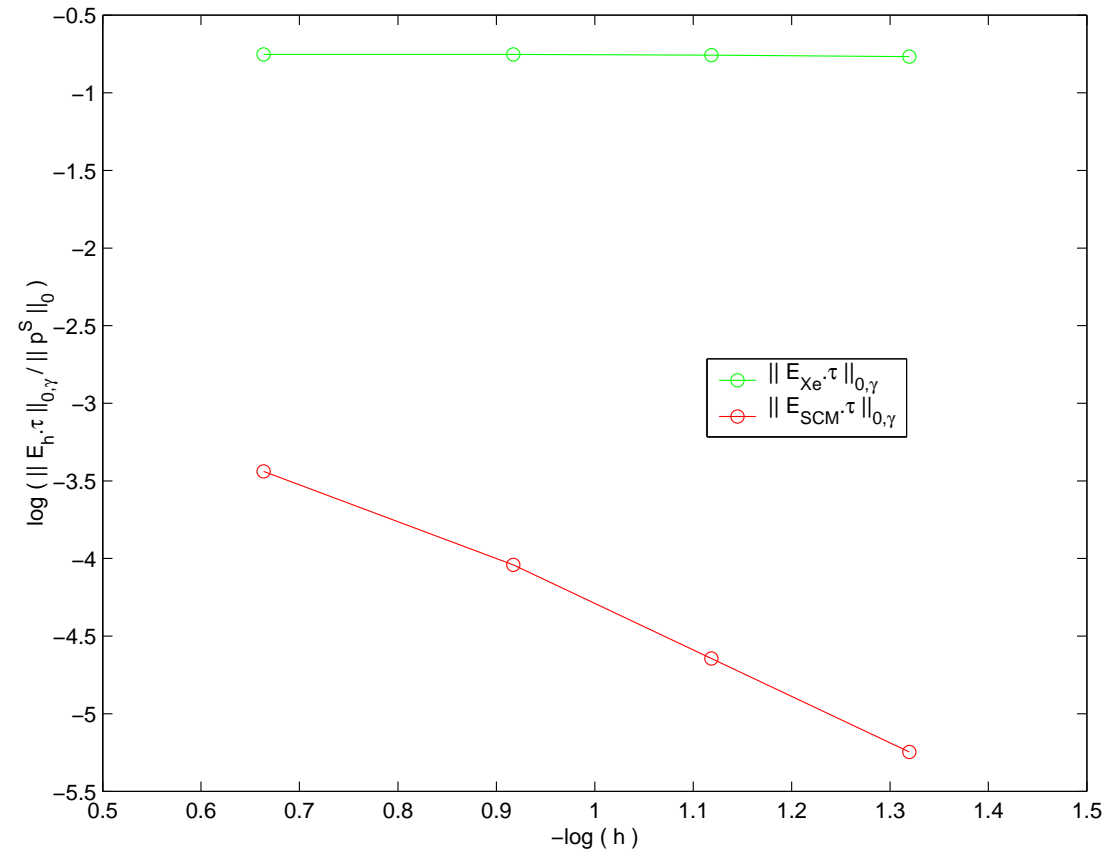
Singular case, $X_{\vec{E}}$ norms

● $\operatorname{div} \vec{E} = p^S$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$.



Singular case, $L^2(\gamma)$ norms

• $\operatorname{div} \vec{E} = p^S$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_\gamma = 0$.



Singular case, tangential component of \vec{E} on γ

● $\operatorname{div} \vec{E} = p^S$, $\operatorname{curl} \vec{E} = 0$, $\vec{E} \cdot \vec{\tau}|_{\gamma} = 0$.

$\|\vec{E}_{X_{\vec{E}}} \cdot \vec{\tau}\|_{0,\gamma} = 3.31 \times 10^{-1}$, $\|\vec{E}_{X_{\vec{E}}} \cdot \vec{\tau}\|_{0,\gamma} / \|p^S\|_0 = 1.77 \times 10^{-1}$ on the second mesh.

