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**Inter-Sensor Coupling and Spatial Correlation Effects
on the Capacity of Compact MIMO Antennas**

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Abstract :

In this paper, we try to evaluate the relative influence of mutual coupling and of radiation fields correlation in multi-element antennas (MEA) on the MIMO channel performance. We estimate the channel capacity using three circular arrays of monopoles antenna configurations : a real array, a virtual array, and an array of distant sensors with artificially re-introduced mutual coupling. The two last configurations are considered in order to discriminate between the correlation of radiation fields and that due to mutual coupling, whereas the real array is subjected to both phenomena. We also study the influence of the array size, the number of electromagnetic sensors in the array and the load impedance. It appears clearly that coupling generally enhance capacities for compact arrays. However we also observe capacity reduction resonances due to coupling for certain array sizes. This is explained by degeneracies of the coupling matrices, which are responsible for correlations in transmission or reception.

I – Introduction :

MIMO systems [1] are very interesting in radiocommunications because they can offer high channel capacities. MIMO techniques indeed benefit from the mobile radio channel multipath character and from its underlying multiple sub-channel structure. MIMO communications require MEA at both ends of links to take advantage of antenna diversity [2]. Diversity is an important issue in MIMO performance, which is affected by the correlation coefficients between the transmitted or received signals in the MEA. In order to enhance diversity at both ends of the link and realize high capacities, the antennas need to employ a sufficient number of sufficiently spaced electromagnetic sensors. Indeed a small antenna spacing decreases diversity performance, due to electromagnetic fields correlation. However the antenna size can be utterly inconvenient for antennas which are installed in mobile stations. In addition the mutual coupling also increases for closely spaced sensors, which can detrimentally modify the channel capacity. In this paper, we firstly present the channel model used for capacity computations. We secondly describe the various antenna array configurations used, placing emphasis on the radiation fields correlation, or alternatively on the inter-sensor coupling, in order to discriminate between these two phenomena. Then, we evaluate their respective contributions to the channel capacity behavior. After discussing the simulation results, we finally conclude about the importance of mutual coupling and spatial correlation effects in MIMO antenna design.

II – System Model :

A communication system with n_T antennas at the transmitter and n_R antennas at the receiver is considered :

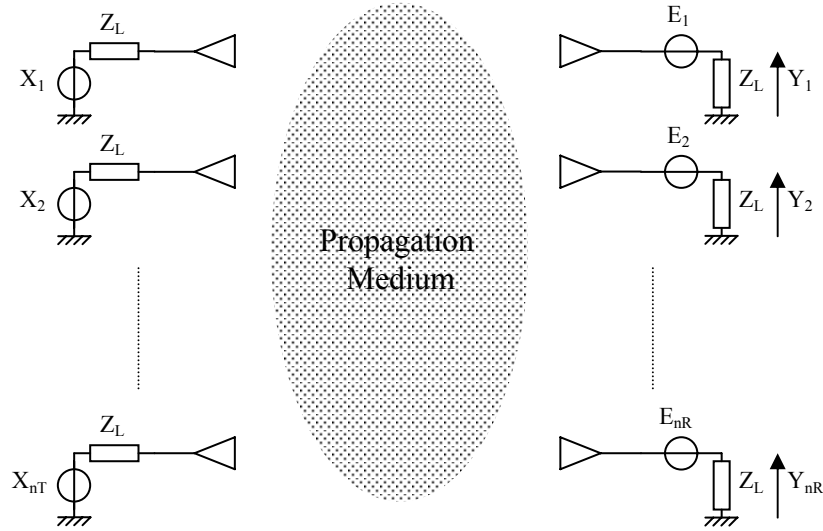


Figure 1 : Ideal MIMO System

Where $X(n_T)$ and $Y(n_R)$ are the transmit signal vector and receive signal vector respectively. $E(n_R)$ is a vector of received open-circuit voltages. Each element antenna is loaded by a real impedance Z_L . The system can be characterized by a channel matrix $[H]$, consisting of n_R lines and n_T columns :

$$H = \begin{bmatrix} h_{1,1} & \dots & \dots & \dots & h_{1,n_T} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & h_{i,j} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ h_{n_R,1} & \dots & \dots & \dots & h_{n_R,n_T} \end{bmatrix}$$

Each element $h_{i,j}$ of $[H]$ is the complex transmission coefficient between the transmit sensor j and the receive sensor i . Thereby, we can deduce the relation between the transmit signals and the received signals :

$$Y(n_R) = H(n_R, n_T)X(n_T) + N(n_R)$$

Here $N(n_R)$ is an added noise vector. In the present work, we use the following normalization condition [3] :

$$\|H(n_R, n_T)\|_F^2 = n_R n_T$$

$\|H\|_F$ denotes the Froebenius norm (the square root of the sum of the squares of all matrix elements) of the matrix $[H]$. This condition actually allows antenna gain at the receiver side to enter the computed capacities [4]. The channel matrix $[H]$ can be subsequently decomposed as :

$$H(n_R, n_T) = A_R(n_R, N)W(N, M)A_T^T(n_T, M)$$

$W(N,M)$ is the wave connecting matrix (WCM) [5] and $A_T(n_T, M)$ and $A_R(n_R, N)$ are the steering matrices at transmitter side and receiver side respectively. M and N are the numbers of rays at the transmitter side and at the receiver side. The superscript T designates the transposition operator. The matrix $[W]$ which connects the arrival waves to the departure waves by complex coefficients is in particular useful to model keyholes. In this paper, this matrix will simply be taken as identity meaning that all waves have the same amplitude. The steering matrices $A_T(n_T, M)$ and $A_R(n_R, N)$ relate the propagating waves to the transmit and to the receive signals. A rich propagation environment can already be obtained with $M=N=10$. In the simulations below, the MEA will consist of n_T or $n_R = 2, 3$ or 4 elements. The directions of departure and arrival (DOD and DOA) are random and uniformly distributed over 360° azimuth, with 0° elevation. To complete the statistical distribution of channel realization, a microlocal statistic of antenna position is introduced by varying the element antenna position on a distance of λ or 2λ on the orthogonal axis X and Y of the horizontal plane. λ being the wavelength. Thereby, the channel capacity is also a random variable and can be obtained by [3] :

$$C = \log_2 \left[\det \left(I_{n_R} + \frac{\rho}{n_T} HH^H \right) \right] \text{ in b/s/Hz}$$

Here ρ is the signal to noise ratio (SNR) which will be kept to 20 dB in all simulations and the superscript H designates the conjugate transposition. I_{n_R} returns the identity matrix of n_R lines and n_R columns. We compute the cumulative distribution function (CDF) of the channel capacity using Monte-Carlo simulation. To determine the MIMO channel performance, we will refer to the mean value of the capacity C . In the presence of mutual coupling the channel matrix $[H]$ can be modified as [6] :

$$H'(n_R, n_T) = C_R(n_R, n_R)H(n_R, n_T)C_T(n_T, n_T)$$

And the MIMO system becomes :

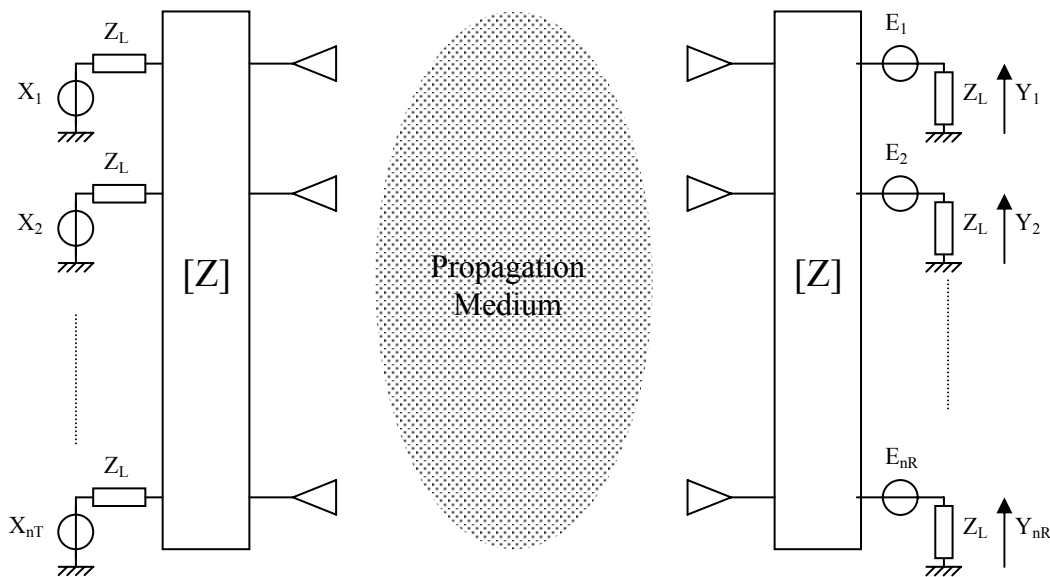


Figure 2 : MIMO System in Presence of Mutual Coupling

$C_T(n_T, n_T)$ and $C_R(n_R, n_R)$ are the coupling matrix at transmitter side and receiver side respectively. Each element $C_{i,j}$ of $[C_R]$ is a complex coefficient relating the true voltage at element i to that which would be obtained in the absence of coupling (receiving mode). An analogous interpretation exists in transmission for the radiated fields instead of the voltages. These matrices depend upon the electromagnetic phenomena inside the antenna array (modeled by a mutual impedance matrix $[Z]$) and the load impedances Z_L connected to the element antennas.

III – Array Antenna Configurations :

Let us consider different circular array topologies of monopoles which are defined by the number of elements n_T or $n_R = 2, 3$ or 4 , the circle diameter d and the load impedance Z_L :

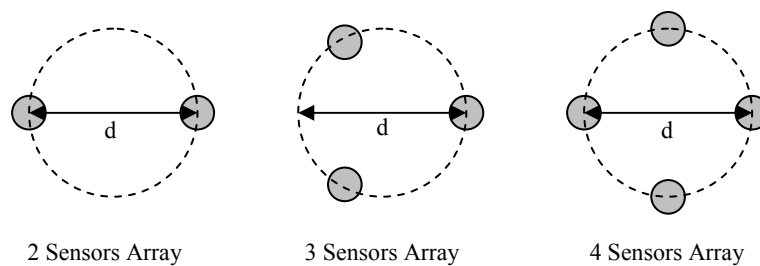


Figure 3 : Circular Array Topologies

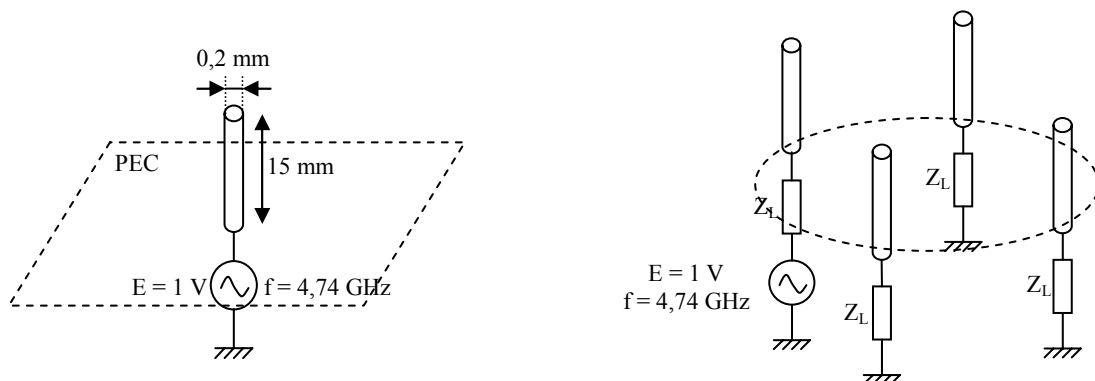


Figure 4 : Simulated Monopole Environment Figure 5 : Simulated Antenna Array Structure

The length of the monopoles is 15 mm corresponding to a quarter wavelength resonance around 5 GHz. In practice the simulated resonance frequency is 4.74 GHz because of monopole thickness (diameter 0.2 mm). Since the impedance of the monopole is real and equal to $\approx 36 \Omega$ at this frequency, we will keep the load impedance to the value $Z_L = 36 \Omega$ unless stated (impedance matching). For a n elements array, a number n of radiation patterns is computed by exciting only one element at a given time. This real array includes both correlations between the radiated or incident electromagnetic fields, and mutual coupling effects, in its diversity properties. In order to discriminate between these two phenomena, we first introduce a virtual array which places emphasis on the spatial correlation of the radiation fields. The array radiation patterns is obtained by computing the pattern of a single monopole on an infinite ground plane (PEC : Perfect Electric Conductor), which is duplicated at different positions taking into account the steering vector. The mutual coupling is trivially absent in this type of configuration. The second array configuration places emphasis on the

mutual coupling, therefore we consider an antenna array where the elements are far away (array diameter of 200 mm) so that spatial correlation and mutual coupling could both be neglected. We then artificially introduce mutual coupling by replacing the matrix $[Z]$ of this array by the impedance matrix $[Z]$ of a small array where mutual coupling is strong.

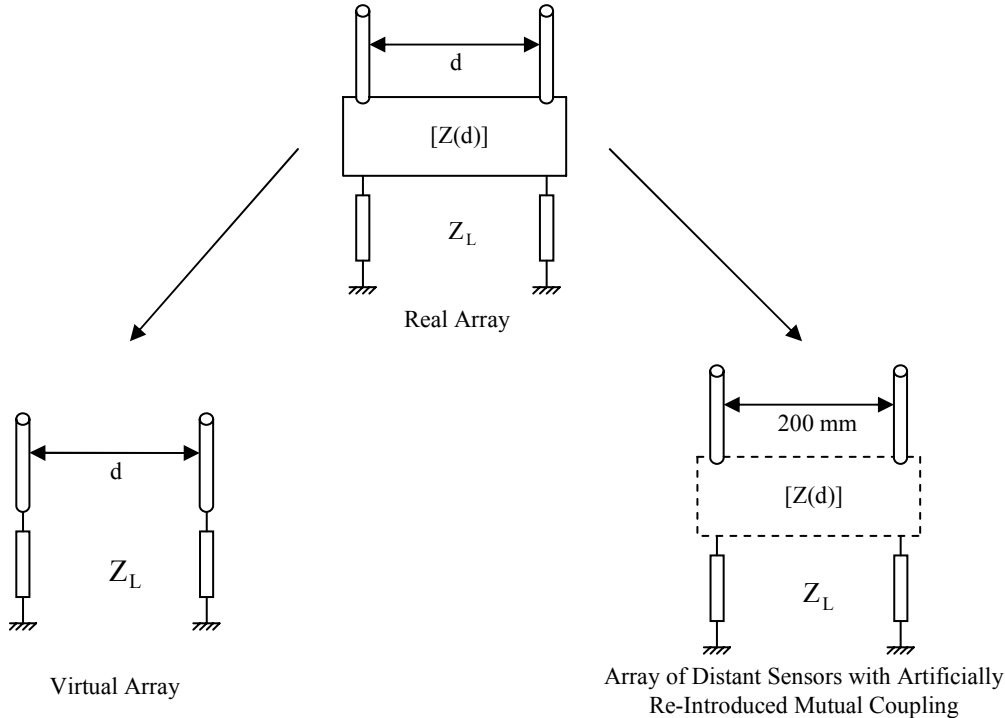


Figure 6 : Array Types

Then, we compute the excitation currents I_i at each monopole element and we apply these currents to the monopole on an infinite ground plane (PEC) to obtain the radiation pattern.

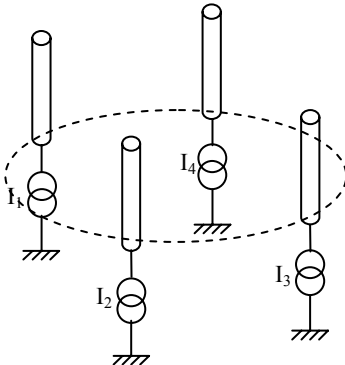


Figure 7 : Monopoles Excitation

IV – Simulation Results :

In all simulations presented here, exactly identical MEA array are used on the receive and transmit side, whatever the array type (real, virtual, artificially coupled). The array complex gain patterns and impedance matrices were in all cases computed “exactly” by a moment method based commercial tool (WIPL).

IV-1 – Real Array :

The evolution of the capacity of MIMO systems using 2, 3 and 4 elements array with the array diameter d is shown below. Here d is varying from 0.4 mm to 100 mm ($0.0063 < d/\lambda < 1.58$). We obtain the following curves of mean capacity :

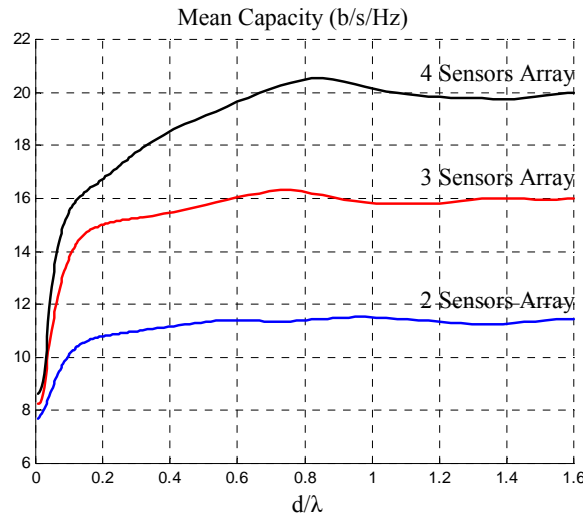


Figure 8 : MIMO Capacity with Real Array

The mean capacity with a n elements array is naturally always greater than for an array consisting of less than n elements. It increases with d/λ in this range : $0.0063 < d/\lambda < 0.6$, beyond which small oscillations appear. The shape of this curves results from both spatial correlation and mutual coupling phenomena.

IV-2 – Virtual Array :

When the mutual coupling is suppressed, the maximum capacities achieved for a given number of sensors are not really changed. Indeed, while mutual coupling is dominating in small array where the elements are closely spaced, it is negligible in the large array limit where the real array does not differ from the virtual array. We can observe the following curves of mean capacity :

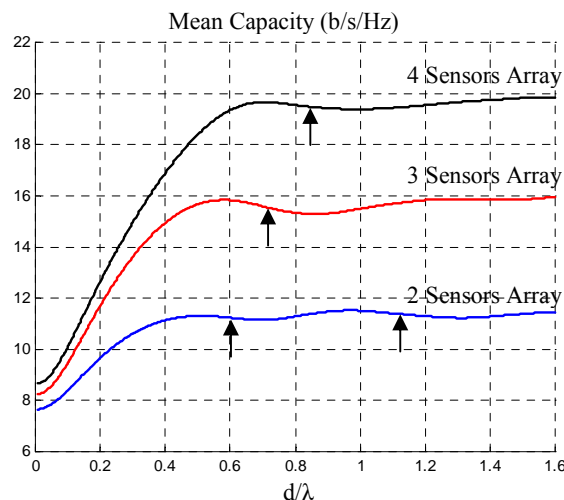


Figure 9 : MIMO Capacity with Virtual Array

We can notice that the diameter d corresponding to the saturation value of the mean capacity is increasing with the number of sensors. Beyond the saturating diameter, small oscillations are also seen. Here these oscillations can be attributed to correlations of the electromagnetic fields [7]. If the probability density function (PDF) of the DOA is uniform over $[0, 2\pi]$, the envelope correlation coefficient between two signals received by two identical antennas is :

$$\Gamma(s/\lambda) = |J_0(2\pi s/\lambda)|^2$$

Where J_0 designates the Bessel function of the first kind and zero order. s is the antenna separation. In the case of two sensors array, s corresponds to the diameter d of the array. The correlation coefficient between the two received signal as a function of antenna separation is :

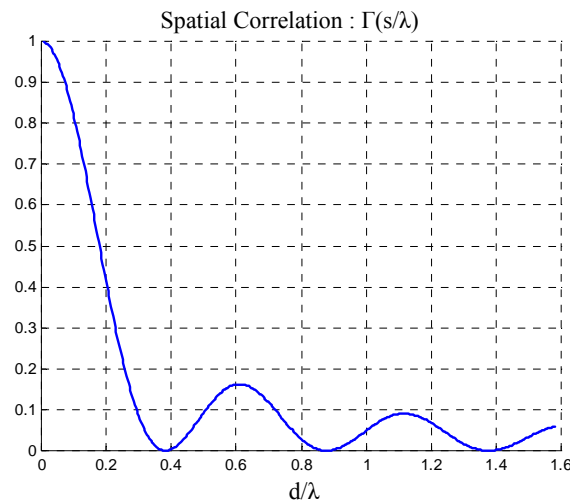


Figure 10 : Correlation Coefficient vs Antenna Separation

Each local maximum of the correlation coefficient graph can be attributed to a minimum of the mean capacity graph of the two sensors array, although the corresponding d/λ values are slightly different. In the case of three and four sensors arrays, these minima also appear when two neighbouring elements are spaced by a distance corresponding to a peak of the above function. The theoretical array diameters are marked on Figure 9. It can be seen that they are systematically smaller than the capacity minima. One possible explanation is that the limited number of waves ($M = N = 10$) in effect reduces the angle spread compared to a perfectly omnidirectional scenario, thus pushing capacity minima to higher diameter values.

IV-3 – Array of Distant Sensors with Artificially Re-Introduced Mutual Coupling :

The array diameter is kept to 2000 mm. The quantity d/λ characterize the mutual coupling introduced by replacing the array impedance matrix $[Z]$ by the impedance matrix of the real array of diameter d . The mutual coupling influence on MIMO capacity can be described by the following curves :

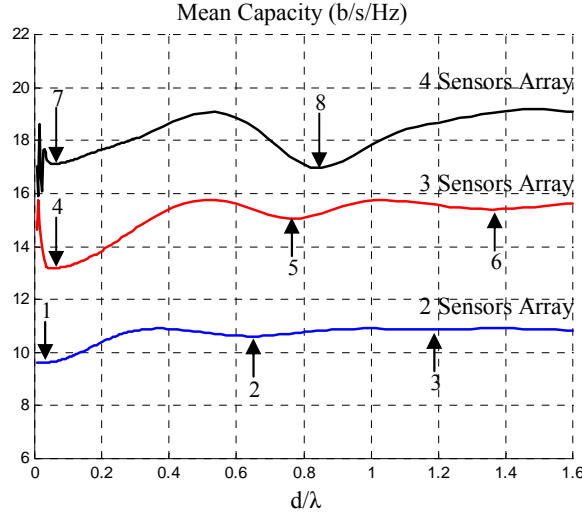


Figure 11 : MIMO Capacity with Artificially Coupled Array

The maximal capacities are also unchanged as expected. Many resonances numbered from 1 to 8 appear at several values of d/λ . In order to understand the presence of these resonances, we developed the following model : let I_j be the complex excitation current at antenna element 1 *operating in transmission mode*. We can evaluate their inter-dependence by analysing the ratio I_2/I_1 in the case of a 2 and a 3 sensors array, and the two ratios I_2/I_1 and I_3/I_1 in the case of a 4 sensors array ¹.

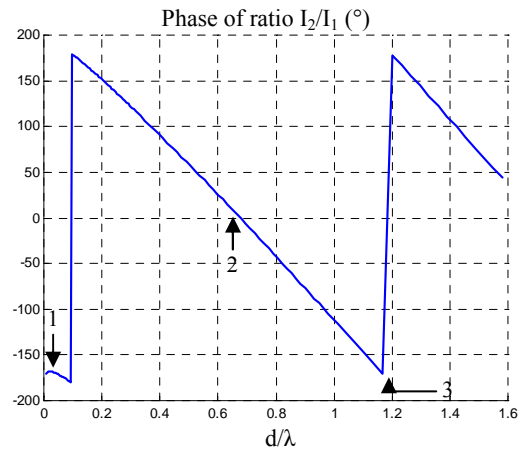
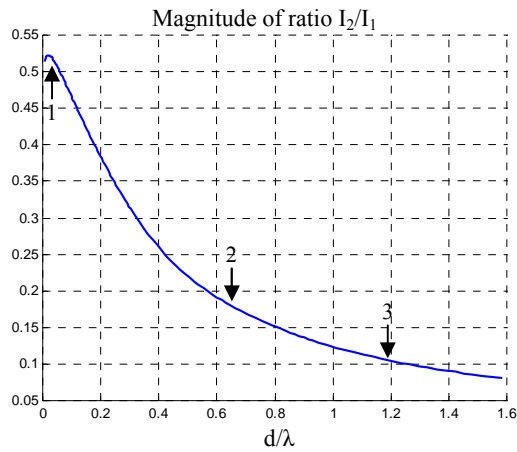
The currents I_j can be precisely computed by solving the matrix equation :

$$[Z + \text{diag}(Z_L)]I = E$$

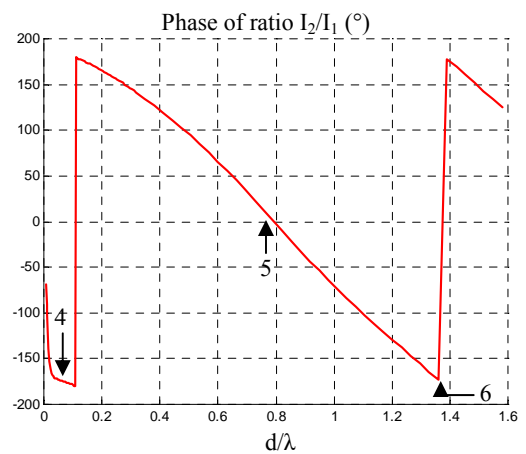
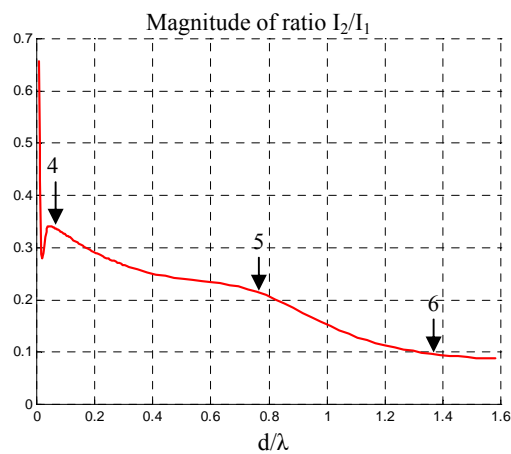
with $[E]$ the excitation voltage vector and $[I]$ the induced current vector. $\text{diag}(Z_L)$ returns a diagonal matrix where the diagonal elements are equal to the load impedance Z_L . Here $[E]$ will play the role of the transmitted signal vector X (Figure 1). Since the currents determine radiation by equivalent isolated monopoles, $Z_L [Z + \text{diag}(Z_L)]^{-1}$ plays the role of the coupling matrix C_T . This is exactly the same for C_R , since $[E]$ is replaced by the *emf* of the antenna operating in reception, and the received signal vector Y is $Z_L [I]$.

In transmission, we excite only the antenna element 1 : $E_1 = 1$ V and $E_j = 0$ V ($j \neq 1$). The evolution of the ratio I_j/I_1 is here :

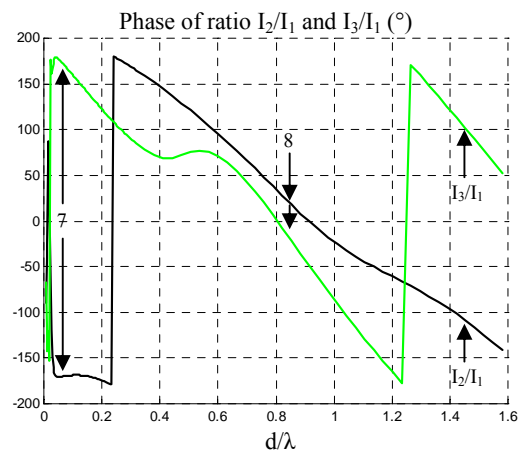
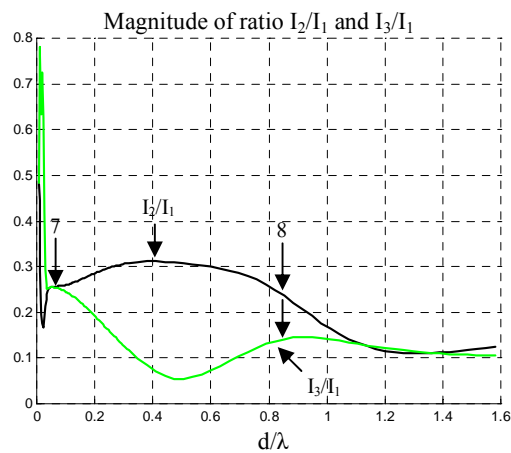
¹ - For a 3 sensors array, we indeed have $U_2/U_1 = U_3/U_1$ in transmission due to the array symmetry. For a 4 sensors array, we have $U_2/U_1 = U_4/U_1$.



Case of Two Sensors Array



Case of Three Sensors Array



Case of Four Sensors Array

Figure 12 : Evolution of Ratios I_i/I_1

We can notice a resonance appears when the phase of one of these ratios approaches 0° or $\pm 180^\circ$ e.g. two currents are phased or opposite phased. These two cases correspond to a channel matrix rank reduction.

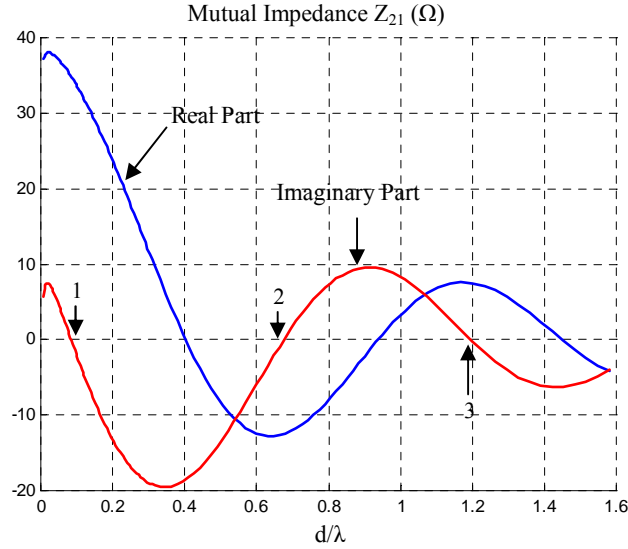


Figure 13 : Mutual Impedance in Two Sensors Array

In the case of two sensors array, these resonances appear when the mutual impedance Z_{21} is a real number. The alternating sign of Z_{21} then determines the phase 0° or 180° of I_2/I_1 . The ratio I_2/I_1 can be expressed by the elements $Z_{i,j}$ of $[Z]$ and the load impedance Z_L :

$$\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_L + Z_{22}}$$

Because the monopole impedance and the load impedance are real, the phase of I_2/I_1 depends only on the phase of Z_{21} . We have seen that channel matrix in presence of mutual coupling could be written [6] :

$$H'(n_R, n_T) = C_R(n_R, n_R)H(n_R, n_T)C_T(n_T, n_T)$$

The determinant of $[H']$ thus involves the product of the determinants of the two coupling matrices $[C_T]$ and $[C_R]$. These matrices contain the inter-relations between the various voltages. In the case of two sensors array, if $U_2/U_1 = 1$ (total correlation), the corresponding coupling matrix is :

$$C_T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

If $U_2/U_1 = -1$ (opposite phased), the coupling matrix is :

$$C_T = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

In both cases, the determinants equal 0. For a 4 sensors array, it appears twice more resonances than a 2 or 3 sensors array because two independent ratios I_2/I_1 and I_3/I_1 can detrimentally influence the capacity. The level of the capacity reduction is also clearly linked to the magnitude of these ratios.

The above model has been developed for an array operating in transmission. However it applies to reception as well, since a received voltage on sensor 1 will electrically induce a voltage on the other sensors because of mutual coupling and will also induce rank reduction of the channel matrix. This is confirmed by computing capacity curves vs array diameter with only one of the two arrays closely coupled instead of both, which only reduces the resonance amplitudes but not their positions or the general shape of the curves. Thus electromagnetically coupled MEA play an identical role in transmission and reception.

IV-4 – Influence of the load impedance (Real array) :

We computed the capacity versus d/λ curves for different load impedances varying from 0Ω to 50Ω :

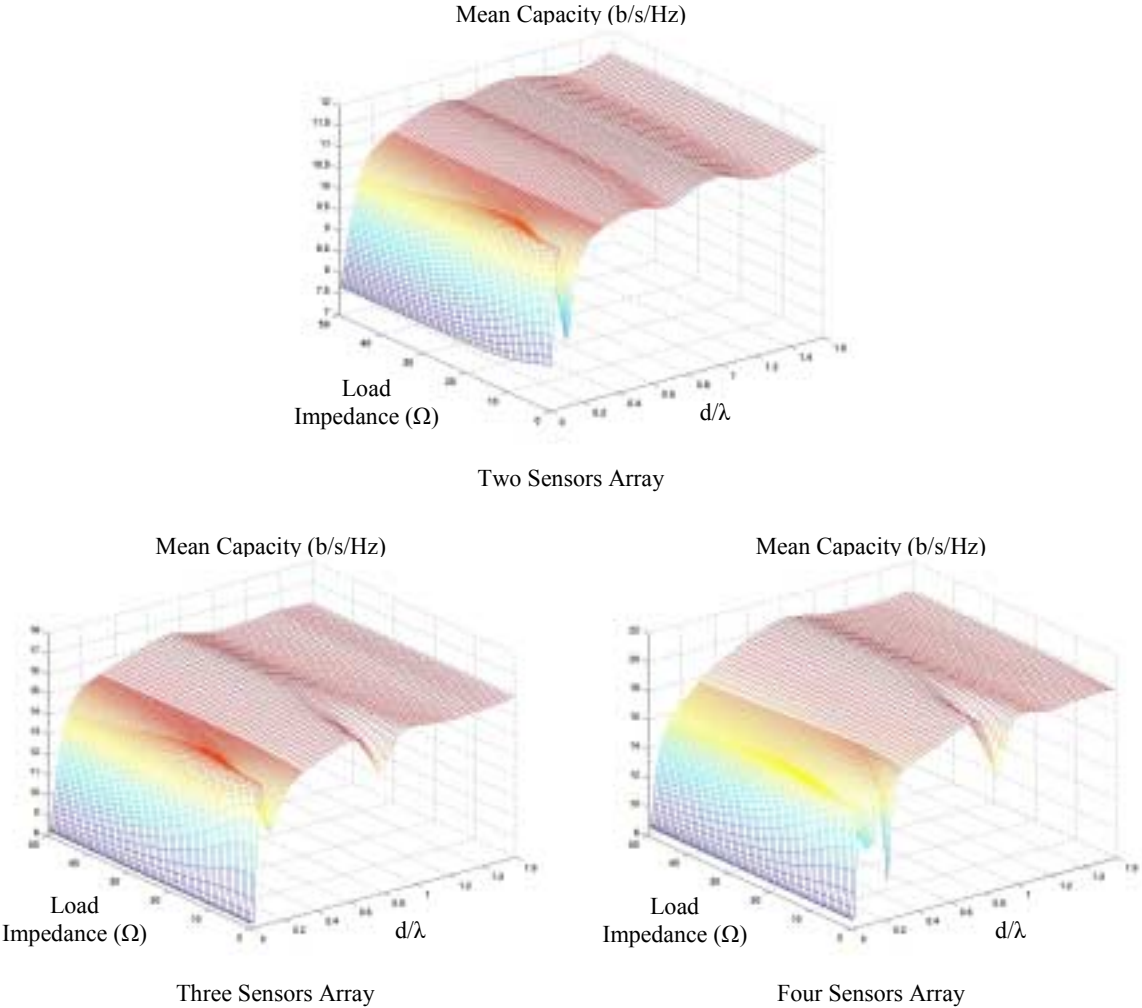


Figure 14 : Influence of Load Impedance (Real Array)

The curves of the virtual arrays are obtained if the load impedance is very high (\approx open circuit). Indeed, in this case the induced currents due to mutual coupling are low. Many resonances are on the other hand present in the case of a null impedance (short circuit). They have the same origin as for the artificially coupling array. Spatial correlation is also present but mutual coupling is dominant at low impedance values.

V – Conclusion :

To conclude, the spatial correlation has a detrimental influence on channel capacity which can be countered by choosing a sufficient array diameter. The adequate size depends on the number of elements. For a large number, the dimensions of the antenna array have to be large. In small arrays, fields correlation has always a detrimental effect on MIMO capacity, but mutual coupling does not always badly affect the capacity. Contrary to common belief, the mutual coupling decreases the correlation between antenna elements, by creating pattern diversity [8] which increases the channel capacity. To preserve mutual coupling without capacity reduction resonance, the load impedance must be large enough; it appears that impedance matching is adequate.

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